BELLCOMM, INC. 955 L'ENFANT PLAZA NORTH, S.W.

WASHINGTON, D. C. 20024

SUBJECT:

Optimization of Hybrid Trajectories for the Apollo Mission Under a DPS Abort Constraint - Case 310 DATE: February 7, 1969

FROM: R. A. Bass

ABSTRACT

An optimization procedure for hybrid missions is presented with some representative results. The hybrid profile differs from the nominal free return profile during translunar flight, with translunar injection (TLI) targeted to a high periselene free return followed by a transfer to non-free return five to ten hours past TLI. All non-free return trajectories considered are constrained such that the Lunar module DPS engine can abort the mission to safe earth return a short time after perilune in the event of Service Module SPS engine failure. Optimization of hybrid trajectories requires consideration of new independent parameters in addition to those associated with the free return mission. The family of free return trajectories defined by perilune distance and orbit inclination must be searched, and the best non free trajectory to couple with each member of the family must be found. Each of the added optimization parameters and constraints is examined for its relative effect on mission propellant economy.

(NASA-CR-103939) OPTIMIZATION OF HYBRID

TRAJECTORIES FOR THE APOLLO MISSION UNDER A

DPS ABORT CONSTRAINT (Bellcomm, Inc.) 30 P

(Bellcomm, Inc.) 30 P

(CATEGORY)

N79-71826

SUBJECT: Optimization of Hybrid Trajectories for the Apollo Mission Under a DPS Abort Constraint - Case 310

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MEMORANDUM FOR FILE

1. Introduction

Hybrid missions have been proposed for later Apollo missions as a compromise between the safety of a free return mission and the propellant economy of the non-free return trajectory. These propellant economies can be used to land at lunar sites out of the Apollo region, to obtain increased payload or to provide a daylight launch for the present Apollo sites.

The hybrid mission differs from the nominal Apollo mission only in translunar flight. The spacecraft would be injected from earth orbit into a high periselene (500 to 5,00 nautical miles) free return circumlunar trajectory followed sometime later by a transfer using the SPS engine into a non-free return trajectory with an optimum periselene altitude for the chosen lunar parking orbit. The resulting non-free return trajectory is constrained by a requirement of abort capability by the LM DPS onto an earth return trajectory a short time (usually 2 hours) after nominal lunar orbit insertion(LOI). The obvious safety advantage over injection into a non-free return trajectory is that, upon initial SPS engine failure, the spacecraft would continue its free return course around the moon and return to earth. The propellant economies arise because the spacecraft requires less AV at lunar orbit insertion as both the translunar flight energy and the plane change requirements at LOI are lower without the free return constraint. The second safety feature of the hybrid mission is the capability to use the DPS engine for an abort from the non-free return trajectory should the SPS engine fail at lunar orbit insertion.

The hybrid optimization program was written to be used in conjunction with the Bellcomm Apollo Simulation Program (BCMASP). Patched conics are used to compute the trajectories involved, with the patching occurring at the moon's sphere of influence (MSI). The hybrid simulation program is described in detail in Reference 1 with appropriate flow charts and program listings. Here attention will be directed to a general discussion of the logic used in the optimization and the influence of the various independent variables on the hybrid trajectory.

2. Description of the Hybrid Trajectory

The basic geometry of the hybrid trajectory is illustrated in Figure 1. The spacecraft is launched, placed into earth orbit and injected onto a free return circumlunar trajectory with a high periselene distance, \mathbf{R}_p . At time, \mathbf{T}_H , after translunar injection the SPS engine is used to transfer the spacecraft onto a non-free return trajectory with an optimum periselene, \mathbf{r}_p . The velocity change associated with the hybrid maneuver is referred to as $\Delta \mathbf{V}_H$. At periselene the spacecraft is deboosted into lunar orbit and proceeds thereafter as in a nominal Apollo mission. Should the SPS engine fail at lunar orbit injection, the required abort maneuver is performed with the Lunar Module DPS.

The transearth legs of the free return trajectory and of the post periselene abort maneuver are restricted to posigrade reentry into earth atmosphere (inclination less than 90° relative to the equator). Spacecraft movement around the moon is retrograde. All constraints unless otherwise noted follow those for the nominal Apollo mission.

3. DPS Abort Capability

Before proceeding with the discussion of the hybrid simulator some comments on the DPS abort capability are in order. As an added safeguard against SPS failure at lunar orbit insertion all hybrid trajectories constructed in the simulator are forced to meet a DPS abort constraint that will allow the DPS engine to change the spacecraft's trajectory after periselene to establish safe earth reentry.

Under the present Apollo weights and propellant capacities, the DPS engine can provide approximately a 2,000 ft/sec ΔV for abort purposes.

From a propellant consumption standpoint the most opportune time for aborting would be at periselene. But this would presume prior knowledge that the SPS engine was not going to function at lunar orbit insertion. Since it is more likely that malfunction would be detected during an attempt at LOI, the DPS abort cannot be expected to perform until sometime after periselene. Two hours after periselene was selected as a reasonable time lapse and is used in the hybrid simulator to determine the abort requirements. The details of the abort calculations are presented in Reference 2.

4. Optimization Philosophy

The optimization objective is to minimize the total mission SPS propellant requirements. The low energy, high periselene free return trajectories require less ΔV at translunar injection but launch vehicle energy requirements were not considered in the optimization.

The optimization logic can be separated into two phases. The inner phase determines the optimum non-free return trajectory that can be flown from the pre-hybrid maneuver state point. The outer phase selects the optimum initial free return high periselene trajectory. Figure 2 is a schematic showing the relationship of the two phases. An optimum non-free return leg is developed for each free return trajectory considered in the outer optimization loop.

The outer phase, circumlunar free return trajectories, under the Apollo reentry constraints, have two degrees of freedom:

- . Inclination of the transearth trajectory plane
- . Periselene distance

Specification of both of these uniquely specifies a circumlunar free return trajectory for a given launch azimuth and injection type. Apollo specifications require that earth reentry must be posigrade so the return inclination must be less than 90° relative to the earth's equator. Periselene distance is bounded on the low end by the minimum safe distance the spacecraft can pass over the lunar surface.

The inner phase must determine the best non-free return trajectory starting from a specified position in space. Here, three parameters are available for optimizing the mission.

- the inclination of the lunar approach hyperbola
- . the time of flight from the hybrid maneuver to LOI
- . the orientation of the lunar orbit

The values of these three independent variables must be chosen such that the DPS abort ΔV required to return to earth is less than a specified maximum as well as for minimum SPS propellant consumption. The coast time between TLI and the hybrid maneuver could also be added as an independent variable but it was found

that the sensitivity of propellant consumption to the time of hybrid maneuver was not high. A coast time of 5 hours was used for the hybrid simulator. Shorter times may not be feasible from an operational standpoint.

Minimization of the SPS propellant consumption during the mission is accomplished in the optimization program by maximizing the remaining spacecraft weight after transearth injection. This weight is referred to as the "Payoff". Payoff is determined by the propellant requirements for four velocity changes performed by the CSM. In chronological order they are:

- . Hybrid maneuver ΔV_{H}
- . Lunar orbit insertion $\Delta V_{
 m LOT}$
- . CSM plane change maneuver prior to LM ascent $\Delta V_{\mbox{\footnotesize{PC}}}$
- . Transearth injection $\Delta V_{
 m TEI}$

The relative influence of each ΔV is discussed in relation to the five optimization parameters.

5. Design of the Inner Phase

For convenience in the optimization program the second variable, time of flight, is replaced by the translunar trajectory energy, $\mathbf{E}_{H}.$ This is the energy of the spacecraft as it travels the elliptical path from the hybrid maneuver to the MSI. The third optimization parameter, the orientation of the lunar orbit, is expressed in terms of the flight azimuth, $\alpha_{23},$ as the spacecraft passes over the lunar landing site.

The DPS abort constraint is imposed on the ${\rm E}_{\rm H}$ loop so all profiles considered in outer loops meet this constraint. The hierarchy of the iteration loops for both the inner and outer phases is presented schematically in Figure 3. The nomenclature is explained in the following sections.

The optimization logic will be described from the inner most loop outward. (Refer to Figure 3). It must be remembered that optimization parameters in loops outside the one under discussion are constant at either the assumed values at the beginning of the program or values under consideration by the logic of the outer loops.

Approach Hyperbola Optimization

Of the three independent variables used in optimizing the non-free return leg, the inclination of the trajectory from the hybrid maneuver to LOI is optimized first. Within the moon's sphere of influence the translunar trajectory is hyperbolic and for a given trajectory energy a family of hyperbolas exist that intersect the desired lunar parking orbit and also have a periselene that will not cause impact with the lunar surface. (The periselene is constrained to be greater than 40 nautical miles from the surface and less than or equal to the desired lunar parking orbit radius.) The family is searched for the hyperbola yielding the minimum $\Delta V_{\rm LOI}$. Details of the search technique can be found in Reference 3.

Energy Optimization

Figure 4 shows a typical plot of the maneuver ΔV requirements as a function of E_H with the outer three optimization parameters frozen. (α_{10} and R_p are parameters used in the outer phase and are discussed in Section 6.) Abort ΔV is observed to decrease with increasing energy, reaching a minimum near an energy equivalent to that of the free return trajectory. The range of energy considered in developing the optimization program was approximately -6×10^6 ft $^2/\text{sec}^2$ to -11×10^6 ft $^2/\text{sec}^2$. Consideration of translunar energies greater than those required for nominal free return missions (i.e., 60 n.m. periselene altitude) are not profitable in terms of payoff. Energies lower than -11×10^6 ft $^2/\text{sec}^2$ require abort ΔV 's that generally exceed Apollo capabilities.

Examination of the variation in the individual ΔV 's is useful in explaining the effect on payoff. The ΔV at transearth injection (TEI) shows little variation as can be seen in Figure 4. The major effect of the translunar energy variations on TEI is to change the time at which TEI occurs. The spacecraft always injects from approximately the same lunar orbit relative to the earth since α_{23} is a constant (in Figure 4). The only significant change in the ΔV_{TEI} is that required to adjust transearth flight time so as to maintain earth landing in the same geographic area (since translunar flight time varies with E_H).

Two small steps in the TEI ΔV curve (Figure 4) are evident at energies near -6.6 and -9.8x10⁶ ft²/sec². This is a result of a nominal Apollo mission constraint on the maximum transearth flight time. As energy is increased the translunar flight time is decreased resulting in an earlier TEI time. By using a longer transearth flight time the space-craft can continue to reach the same earth landing area. So as lunar arrival time decreases, transearth flight time increases until the required transearth flight time exceeds the constraint. In order to meet this constraint the transearth flight time is reduced by about 24 hours and the landing site is reached on the previous earth revolution. This reduced flight time requires higher ΔV at TEI and is seen as the step in the curves.

The lunar orbit insertion (LOI) ΔV generally increases with translunar trajectory energy. Since the lunar orbit is fixed, higher translunar energy means more ΔV is required at insertion. To guarantee an inplane ascent of the lunar module the CSM makes a plane change prior to rendezvous. This CSM plane change ΔV is constant through the energy range since the relationship of the lunar orbit to the landing site is unchanged.

The hybrid maneuver requirement, ΔV_H , minimizes when the non-free return trajectory energy, E_H , is nearly equal that of the high periselene free return trajectory, E_F . As the E_H deviates from E_F , a greater ΔV_H is required to make the change from a free to a non-free return trajectory.

Thus with ΔV_{TEI} and ΔV_{pc} nearly invariant, payoff is driven by ΔV_{LOI} and ΔV_{H} . At energies greater than E_{F} both ΔV_{LOI} and V_{H} are increasing, so payoff will decrease: therefore energies greater than E_{F} generally need not be considered.

The lower bound of E_H is fixed by the minimum energy needed to reach the lunar sphere of influence. Application of the DPS abort constraint generally will increase the lower bound on E_H and reduce the range of E_H available for optimization. Besides E_H , DPS abort ΔV is dependent on the inclination of the pre-abort trajectory and the moon's radial distance from the earth.

As shown in Figure 4, DPS abort ΔV generally varies inversely with E_H up to E_F . If the maximum DPS abort is near 2,000 ft/sec, which is the present Apollo capability, the minimum bound on E_H is set by the abort ΔV requirement. Greater DPS abort capabilities would increase the range of E_H available for optimization.

The problem is to find the energy ${\rm E}_{\rm H}$ with the greatest payoff in the range bounded on the low end by the ${\rm E}_{\rm H}$ associated with the maximum DPS abort and on the high end by the ${\rm E}_{\rm H}$ that is roughly equal to the ${\rm E}_{\rm F}$, the energy of the free return trajectory. The optimization program first determines the ${\rm E}_{\rm H}$ associated with the DPS abort ΔV capability via an iteration process assuming a second order relationship exists between the two. Then ${\rm E}_{\rm H}$ is increased linearly with a fixed step size until payoff shows a decrease. A parabolic fit is then made on payoff versus energy to find the optimum.

Two special cases require comment. Occasionally E_F (i.e. for large values of R_p) is less than the E_H associated with the DPS abort capability. Since both LOI and the Hybrid ΔV increases (and thus payoff decreases) when energy increases beyond E_F and the abort ΔV increases when energy is reduced, the maximum payoff occurs at the energy where DPS abort ΔV equals the maximum allowable. This situation exists for the conditions presented in Figure 5. The value of E_F is near the energy where the hybrid ΔV is at a minimum. For Figure 5 E_F is less than -10 7 ft $^2/\text{sec}^2$. The abort ΔV exceeds the maximum of 2000 fps for energies less than -9.0x10 6 ft $^2/\text{sec}^2$. The maximum payoff trajectory is the one associated with the maximum abort ΔV .

Secondly, cases arise where the abort ΔV requirement never is less than the capability. Here the program uses the energy associated with the minimum abort ΔV and continues the optimization procedure. These situations normally arise for lunar landing sites that are far off the moon's equator.

α_{23} Optimization

The best α_{23} (or lunar parking orbit) is selected from approximately fifteen α_{23} 's determined by consideration

of the landing site latitude and the lunar stay times. The maximum and minimum values of α_{23} are determined within the specified constraints and then the range is broken into equal intervals. An optimum mission is run for each α_{23} . The hybrid program runs a rough cut first using an interval size of three degrees to determine an optimum region; it then follows with a second search with a step size of 0.5 degrees within the optimum region.

A typical relationship between the SPS ΔV 's and α_{23} can be seen in Figure 6. Hybrid, LOI, TEI, Plane Change and abort ΔV 's are plotted versus α_{23} for four values of free return periselene distances. It must be kept in mind that data for each α_{23} has been determined at the optimum translunar energy. For this particular example each trajectory (a trajectory is associated with each pair of α_{23} and R_p values) has an optimum energy at the maximum abort ΔV capability, 2000 fps.

All the ΔV curves vary smoothly except for TEI which has a jump in the region of α_{23} near -100 degrees. This is caused by the maximum transearth flight time constraint as discussed earlier. The effect of R_p variations are discussed in later sections.

6. Design of the Outer Phase

As was mentioned earlier, \underline{two} independent variables are available for optimizing the free return leg: the periselene distance and the inclination of transearth trajectory plane. For convenience the inclination is expressed in terms of the azimuth, α_{10} , of the transearth trajectory (velocity vector) at the exit from the MSI measured positive clockwise from an earth meridian. The limits on azimuth for posigrade reentry are 0 to 180 degrees. The optimization loops for the two parameters are grouped as shown in Figure 3 with R p optimized first. The order is important because of the parabolic fit technique as will be discussed later.

$R_{\rm p}$ Optimization

The periselene distance on the free return trajectory can vary over a wide range. The lower bound is determined by an operational constraint, the minimum safe distance that the

spacecraft would be allowed to pass by the moon, approximately 40 n.m. above the surface. The upper bound is not set by an operational constraint but is practically limited by the abort constraint.

Without an abort constraint, payoff and also translunar flight time would increase with $\rm R_p$. As mentioned before there is a minimum translunar energy associated with the maximum DPS abort ΔV . This minimum hybrid energy is not influenced strongly by variation in $\rm R_p$ as can be seen in Figure 7. Thus the lower bound on hybrid energy remains nearly constant. On the other hand, the upper bound defined by the free return energy, which occurs near the minimum hybrid ΔV point, is decreasing as $\rm R_p$ is increased. When the upper bound on hybrid energy is less than the maximum abort limit, (as represented by Figure 5) then it becomes less than the minimum allowed under the abort constraint, and the hybrid maneuver must increase the velocity sufficiently to bring the energy up to the acceptable level for the abort. The $\Delta V_{\rm H}$ is increased and, concurrently, payoff is reduced.

The optimization program finds hybrid trajectories corresponding to three values of $\mathbf{R}_{\mathbf{p}}$ and fits a parabola through them in search of the best payoff.

α_{10} Optimization

The outer loop of the simulator determines the best inclination to fly the free return leg. Here again the parabola fit technique is employed. A parabola fit is made to three preselected values of α_{10} to determine the optimum. The α_{10} loop is completed last because at larger values of R_p where the DPS abort influence is strong, payoff versus α_{10} for constant R_p shows two maxima while payoff plotted versus α_{10} for the best R_p generally remains unimodal (Figure 12). This is a result of the payoff for a range of α_{10} being depressed as a result of fulfilling the DPS abort requirements. Removing the DPS abort constraint makes the α_{10} curve unimodal also.

Figure 8 through 12 shows the influence of the two outer loop parameters on $\Delta V_{\rm ABORT},~E_{\rm H},~\Delta V_{\rm H},~\Delta V_{\rm LOI},$ and Payoff.

The abort constraint effect at higher values of R_p is seen in Figure 8 where the plateau represents the region of maximum abort ΔV . The abort constraints effect on the other dependent parameter is quite noticeable. Figure 9 shows how E_H also flattens in the same region of R_p and α_{10} as the abort ΔV . The ΔV_H requirements increase significantly (Figure 10) in the same region, as the difference in E_F and E_H increases for higher R_p .

Figure 11 presents $\Delta V_{\rm LOI}$ and shows the stabilization of $\Delta V_{\rm LOI}$ with $E_{\rm H}$ in the abort constrained region. Payoff in turn is depressed for this region due to the increased $\Delta V_{\rm H}$ requirements (Figure 12).

7. Examples

Table A presents a mission profile comparison for four trajectories to Littrow, in July 1970. Littrow is a proposed lunar exploration site located at 22°12'N, 29°20'E, out of the Apollo region. The three profiles compared are:

- . the nominal Apollo free return mission
- . a restricted non-free return profile constrained to meet a 2,000 ft/sec abort ΔV requirement
- . a hybrid mission profile

All missions were flown with the current Apollo weight model, Pacific injection, and a launch azimuth of 90 degrees. For the assumed weight model, an injected weight of 100,862 represents full SPS propellant tanks.

Injected weight requirements for the hybrid are seen to be over 9000 lbs less than the free return mission, approaching that of the non-free return profile. Only a small sacrifice in injected weight is required to gain the safety advantage of the hybrid over the non-free return. The sacrifice is made up almost exclusively from the additional 34 ft/sec ΔV used at the hybrid maneuver. The other ΔV 's, LOI and TEI, are nearly equal to the non-free return profile. The CSM plane change is slightly improved with the hybrid. The translunar flight times are quite close and exceed the free return flight time by around twenty hours. In other respects the hybrid and non-free return missions are similar.

A second example demonstrates that daylight launches throughout 1969 can be achieved by using hybrid trajectories in the latter part of 1969. The attached chart, Table B, shows the injected weight requirements, launch time and translunar flight time for free return (Pacific and Atlantic injection) and hybrid (Pacific injection). The injected weight data is based on the December, 1968 spacecraft inert weights for Apollo 11. Two lunar sites in the west were considered during the last five months for 1969. The western sites were selected for this analysis since hybrid lunar accessibility is most restricted in the west.

The hybrid trajectories allow a daylight launch (Pacific injection and the best of the two sites) with injected weight varying over range of plus 1463 lbs to minus 1468 lbs compared with the nominal free return with night launches (Atlantic injection) for all five months shown. The highest injected weight resulting was 99,408 lbs.

Using hybrid to obtain daylight launch and the southern (W1) site for the entire period would result in injection weight changes from +3260 to -250 pounds. The highest injected weight result was 100,689 lbs. The translunar flight time was increased from 12 to 20 hours with the hybrid.

The hybrid trajectories were determined with the midcourse maneuver, transferring the spacecraft from the high-periselene free return trajectory to the nominal periselene non-free return trajectory, at five hours after TLI. All the hybrid trajectories used meet a DPS abort ΔV constraint of 2000 ft/sec targeted from a point two hours after periselene.

8. Conclusions

A number of general observations concerning the optimized hybrid mission based on preliminary results from the hybrid simulator are listed below.

- the optimum free return periselene will vary from 500 to 5000 n.m. for most missions
- the time of translunar flight for an optimum hybrid mission will exceed the free return flight time by up to twenty hours

- . the hybrid ΔV requirement $(\Delta V_{\mbox{\scriptsize H}})$ at 5 hours past TLI normally is in the vicinity of 30 to 50 fps for an optimized mission
- . hybrid payloads are nearly as high as restricted non-free return payloads with the difference generally attributable to the $\Delta V_{\rm H}$.

2013-RAB-srb

Attachments:
References
Table A
Table B

Figures 1 through 12

BELLCOMM, INC.

REFERENCES

- 1. Caldwell, S. F., "Hybrid Mission Analysis Programs", Case 310, Memorandum for File, to be published.
- 2. Caldwell, S. F. and Hoekstra, T. B., "ABORT A Program Which Determines the Optimum Return-to-Earth Abort Trajectory from Inside the Moon's Sphere of Influence", Case 310, Memorandum for File, to be published.
- 3. Amman, R. J., "Mission Analysis and Open Loop Trajectory Targeting Theory for the Bellcomm Apollo Simulation Program", MM66-4264-2, Technical Memorandum, Bell Telephone Laboratories Inc. January 10, 1966.

LITTROW - JULY, 1970

MISSION PROFILE COMPARISON

	FREE-RETURN	NON-FREE RETURN	HYBRID
Launch Conditions			
Date Time	7/5/70 4:08 pm	7/4/70 4:11 pm	7/4/70 4:12 pm
Translunar			
Free Return Parameters Periselene Radius (n.mi.) \$\alpha_{10}\$ (degrees) Energy (ft^2/sec^2) Flight Time (Hours)	80 37 -6.466x10 ⁶ 70.4		2384 92 -8.96x10 ⁶ (1)
Hybrid AV (ft/sec)	-	3 1 1	34
Non-Free Return Parameters Energy (ft2/sec2) Flight Time (Hours)	! !	-9.24×10 ⁶ 89.7	-9.20x10 ⁶ 90.3
Lunar Orbit			
LOI-AV (ft/sec) Plane change (degrees) ¤23 (degrees)	3386 11.2 -102.4	2648 0.1 -91.3	2647 1.4 -92.0
CSM Plane Change			
ΔV (ft/sec) Angle (degrees)	182 2.0	20	7 0.1

Transearth

TEI-ΔV (ft/sec)	2708	2513	2517
α ₂₈ (degrees)	50.4	104	101
Flight Time (Hours)	110	115	114
Abort (2 hrs. past periselene)			
ΔV (ft/sec)	!	2000	2000
Flight Time (Hours) ⁽²⁾		63.0	63.1
Injected Weight Requirements (lbs)	102670	93454	93661

- (1) The hybrid burn occurs five hours after TLI.
- (2) Flight time is from abort to reentry.

Date

Site - 22.2°N, 29.333°E

Launch Azimuth - 90 degrees

Pacific Injection

Maximum lunar staytime - 44 hours

Spacecraft weights are those current for Apollo 11 as of 12/1/68

TABLE B

HYBRID TRAJECTORIES AS AN ALTERNATIVE TO NIGHT LAIMCH

		HYBKLD TRAJECT(OF	ECTORIES AS AN ALTE OF APOLLO MISSION	AN ALTERNATIVE TO NIG MISSIONS IN 1969	NIGHT LAUNCH	
		XH	HYBRID VERSUS FREE	E RETURN		
		Injection Weight Off	ıt Off Nominal (.(Pounds)	
		Translunar Flight Launch Time	tht Time		.(Hours) .(EST)	
1969	Nort Free Re Pacific	Northern Site (W2) e Return Atlantic	Hybrid Pacific	Sout) Free Re	Southern Site (W1) e Return Atlantic	Hybrid Pacifi
AUGUST	+2097	98322	-1468	1747+	+2160	-250
	63.0 8:51am	65.5 4:07am	78.7 9:50am	61.5 8:46am	62.1 3:57am	75.1 9:35am
SEPTEMBER	+4805	98174	+673	+11425	+3425	+1999
	63.1 11:19am	64.3 3:34am	78.0 10:44am	61.8 9:30am	62.1 3:29am	73.3 10:22am
OCTOBER	+4788	97429	+1463	+9686	+2931	+3260
	65.1 12:20pm	65.2 2:02am	79.8 11:47am	63.1 12:12pm	62.6 1:55am	75.3 11:29am
NOVEMBER	+1852	97556	+515	+6650	+393	+2005
	68.5 2:44pm	66.6 12:22am	85.6 2:23pm	64.3 1:00pm	66.3 12:21am	81.7 2:07pm
DECEMBER	+437	+1771	-345	+2489	96732	+854
	70.1 3:32pm	76.5 10:53pm	87.7 3:13pm	69.3 3:30pm	67.9 10:44pm	85.2 3:03pm

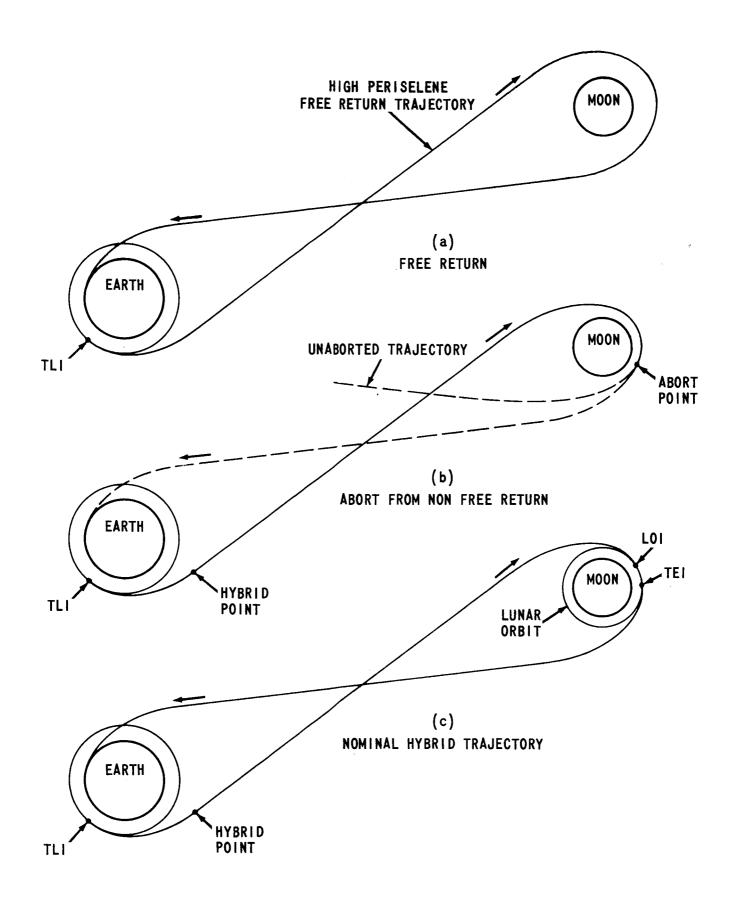


FIGURE I - HYBRID TRAJECTORY DESCRIPTION

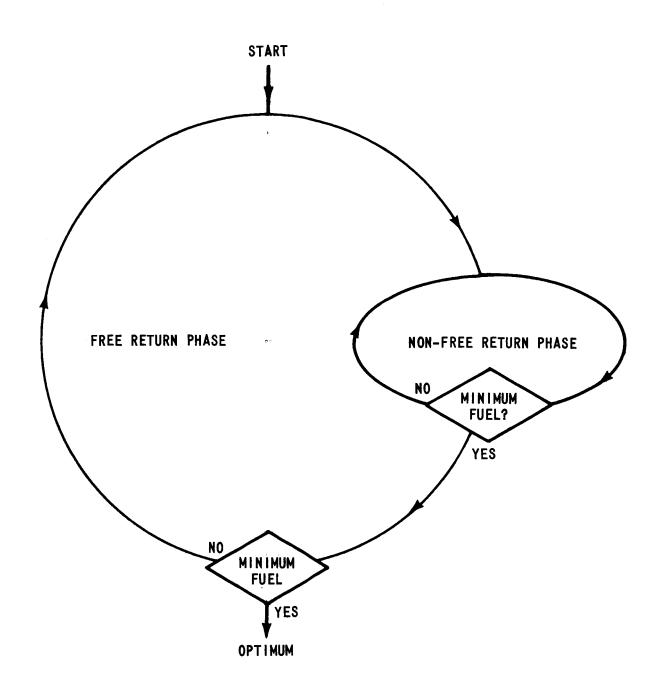


FIGURE 2 - TWO PHASE HYBRID LOGIC

FIGURE 3 - HYBRID SIMULATOR OPTIMIZATION LOGIC

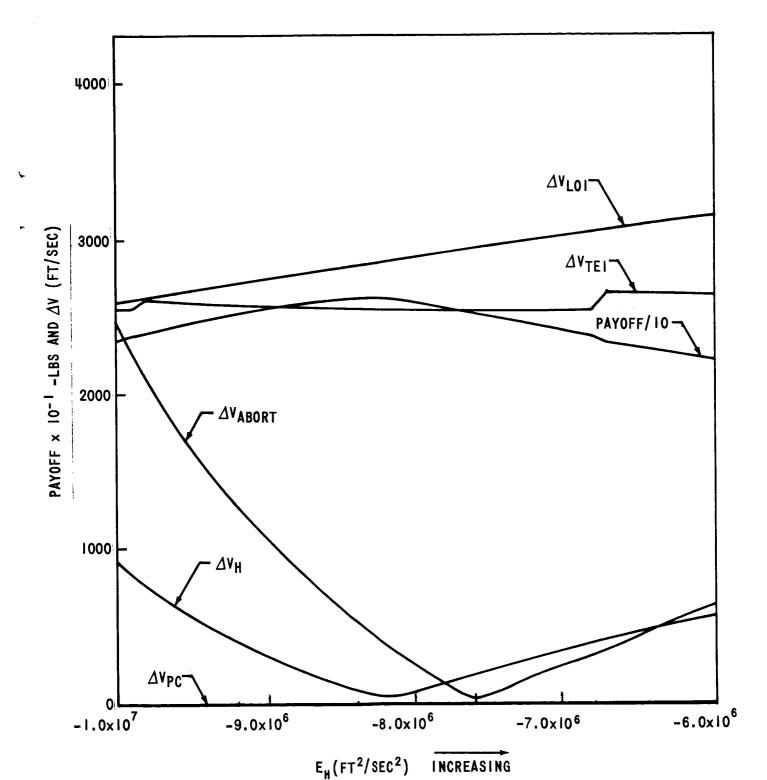


FIGURE 4 - PAYOFF AND Δv AS A FUNCTION OF E_H R_P = 2000 N.M., α_{23} = -90 DEG, α_{10} = 0.0 DEG SITE 0.41667° N., 1.334° W.

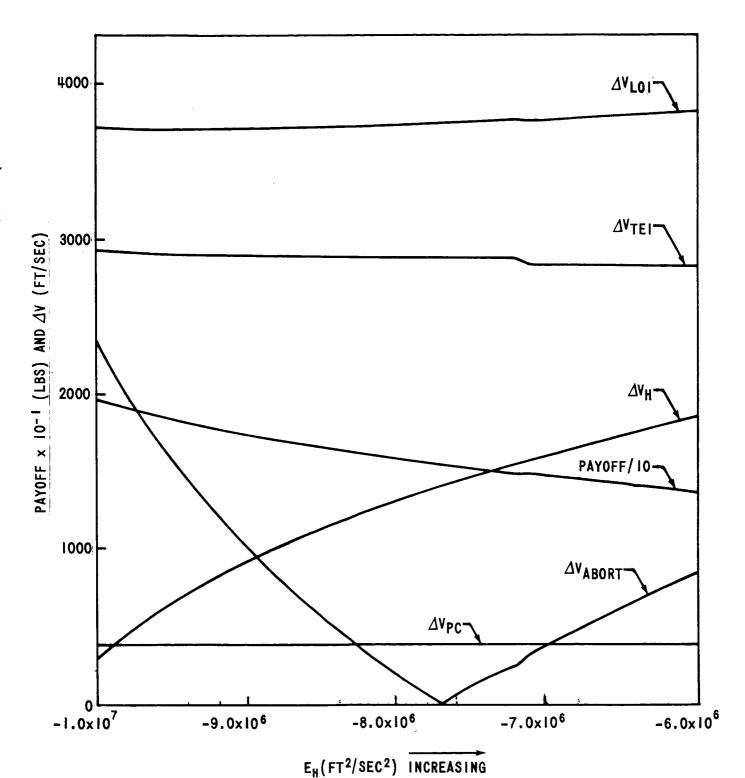


FIGURE 5 - PAYOFF AND ΔV AS A FUNCTION OF E_H R_P = 5000 N.M., α_{23} =-110 DEG, α_{10} = 100 DEG. SITE 0.41667°N., 1.334° W.

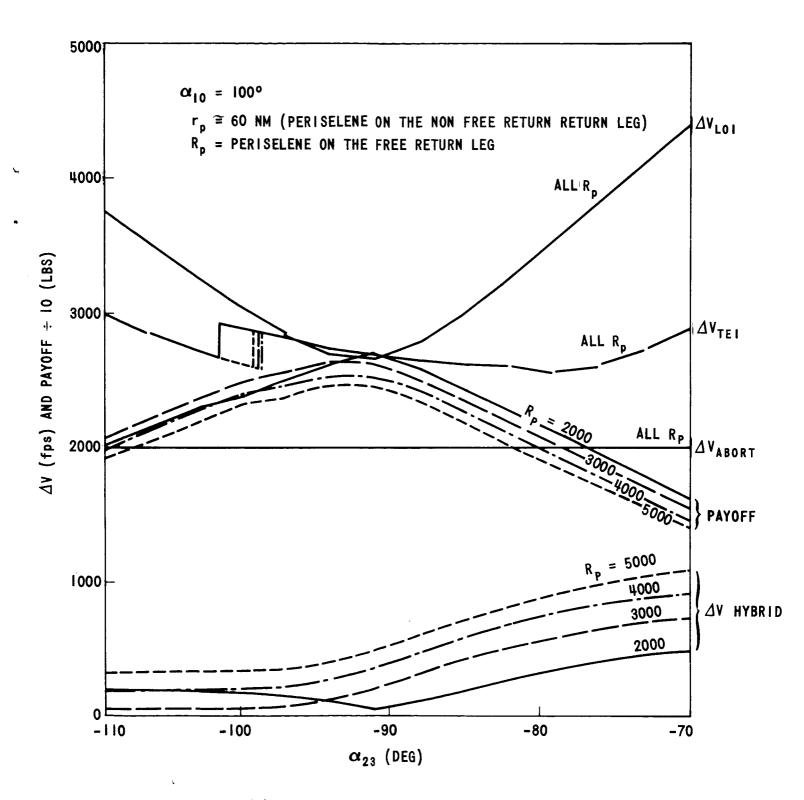


FIGURE 6 - SPS Δ V REQUIREMENTS AS A FUNCTION OF α_{23} FOR VARIOUS FREE RETURN PERISELENE DISTANCES (R_P).

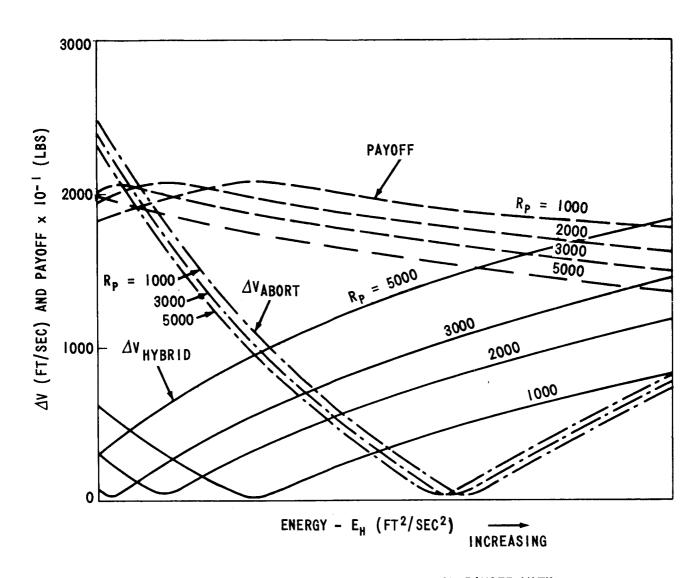


FIGURE 7 - INFLUENCE OF R_p ON PAYOFF WITH α_{10} AND α_{23} INVARIANT.

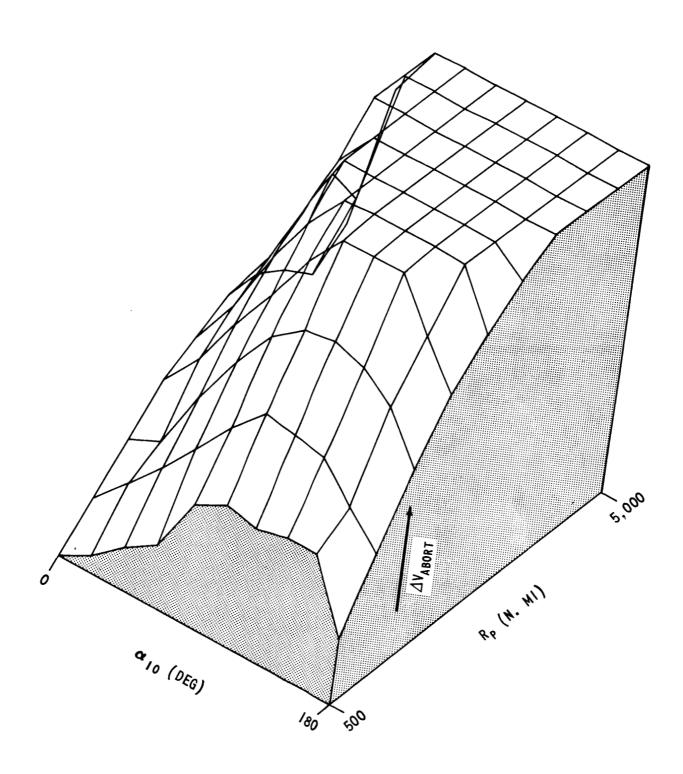


FIGURE 8 - ABORT AV AS INFLUENCED BY THE OUTER PHASE PARAMETERS

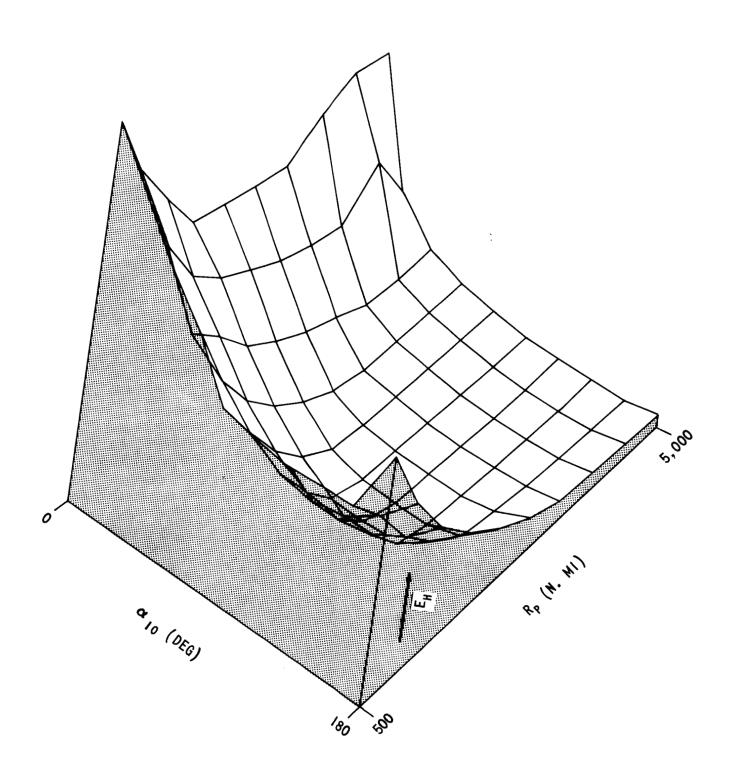


FIGURE 9 - $\mathbf{E}_{\mathbf{H}}$ AS INFLUENCED BY THE OUTER PHASE PARAMETERS

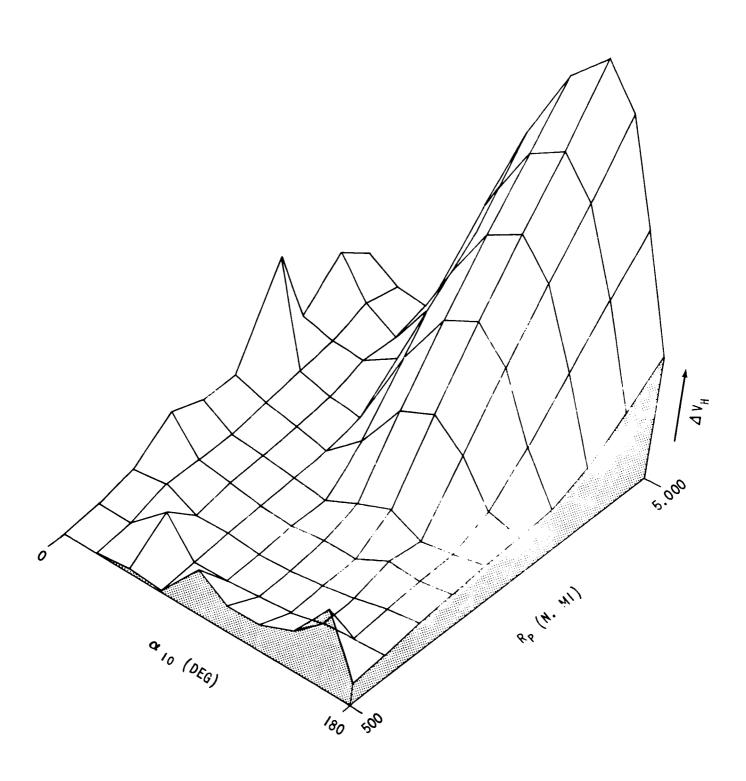


FIGURE 10 - ΔV_H AS INFLUENCED BY THE OUTER PHASE PARAMETERS

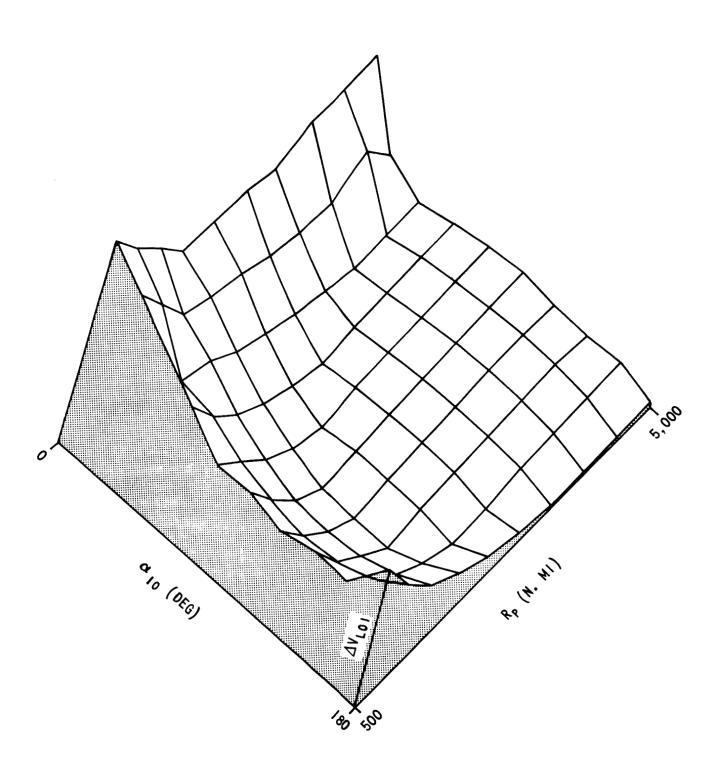


FIGURE II - $\Delta V_{\text{LO}\,\text{I}}$ as influenced by the outer phase parameters

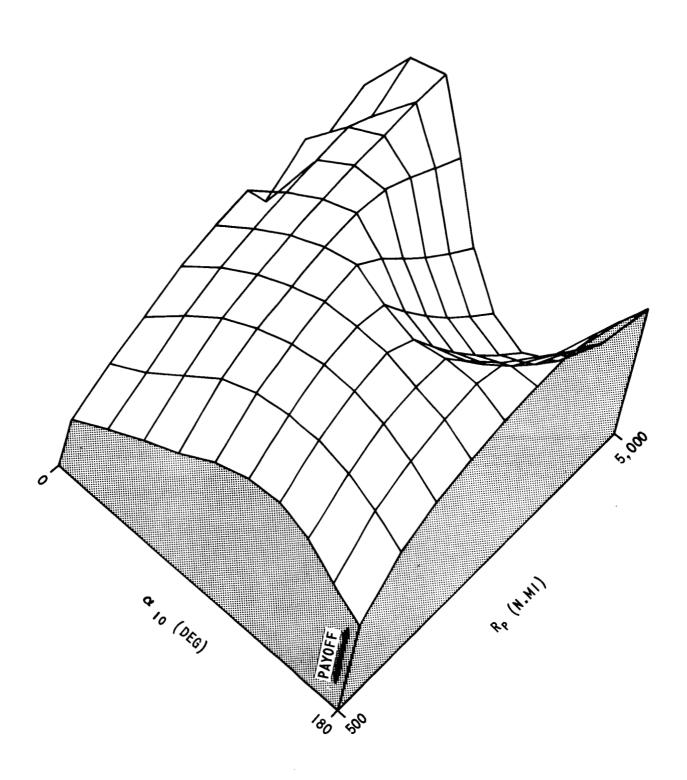


FIGURE 12 - PAYOFF AS INFLUENCED BY THE OUTER PHASE PARAMETERS

BELLCOMM, INC.

Subject: Optimization of Hybrid Trajectories

for the Apollo Mission Under a DPS

Abort Constraint - Case 310

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